

Locating Center of Pressure in 2D Geological Situations

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ABSTRACT

Center of pressure (COP) for horizontal rock slices with realistic density distribution is presented. The location of the COP within the slab depends on the following parameters: (linear) density gradient, compaction constant, density of matrix and that of the pore fluid, and the length and width of the slab. However, no simple proportionality relation amongst the co-ordinates of the COP and these parameters exist. Vertical and thin rock layers such as sedimentary and igneous dykes with different (empirical) relations of spatial density variation can also be worked out in a similar way to locate their COPs.

Key words: Statics, porosity, density distribution, tectonics, structural geology

INTRODUCTION

The center of pressure (COP) is a well-established concept in statics (e.g., Das and Mukherjee 2012). The term sometimes is also used in geosciences in various contexts. For example, the depth of overpressure in a magma chamber is related with the location of the COP within it (Clarke et al., 2007). Tectonic movement and volcanism have been linked with the relocation of the COP (Nishi et al., 2007; also Kumalasari and Srigutomo 2016). Volume and pressure change in a magma reservoir can relocate the COP leading to seismicity (Decker et al., 1983). Exploration for hydrocarbons sometimes, requires drilling to be made close to or at the center of pressure of the reservoir (Chin 2016). However, no direct connection between COP and these geological factors have been explored.

This work makes a preliminary analysis of COP location for a laminar rock slice with geologically realistic density distribution downward. COP for a 3D (irregular) object appears not to be straightforward to deduce (e.g., Bunimovick and Dubinskii 1982) and could be picked up as the follow up detailed work.

THE GEOLOGICAL MODEL

Background

Consider a rectangular lamina/slice of rock immersed in a fluid. We choose X-axis horizontal and the Y-axis vertical, as in Figure 1. In this case the COP will have coordinates (Das and Mukherjee 2012):

$$x = \frac{\int_0^{x_1} \int_0^{y_1} \rho(x,y) x \, dx \, dy}{\int_0^{x_1} \int_0^{y_1} \rho(x,y) \, dx \, dy} \quad (i)$$

$$\text{And, } y = \frac{\int_0^{x_1} \int_0^{y_1} \rho(x,y) y \, dx \, dy}{\int_0^{x_1} \int_0^{y_1} \rho(x,y) \, dx \, dy} \quad (ii)$$

Case 1:

Consider a rectangular parallelepiped and co-ordinate axes as shown in the Figure 1. Density at the origin (0,0) be ρ_0 , and the linear density gradient in perpendicular directions are k_i ($i=x,y$). Mukherjee (2017) reviewed geological cases of crust, lithosphere and sedimentary basins where such density gradients have been reported. Therefore, density variation along X-, Y- and Z-axes are:

$$\rho(x,0) = \rho_0 + k_x x \quad (iii)$$

$$\rho(0,y) = \rho_0 + k_y y \quad (iv)$$

Therefore, for any coordinate (x,y), the density would be given by

$$\rho(x,y) = \rho_0 + k_x x + k_y y \quad (v)$$

Note, putting $x=0$ and $y=0$ in two separate cases, one can go back to eqns (iii) and (iv), respectively.

Putting the expression of $\rho(x,y)$ into eqns (i) and (ii) and performing the definite integral in the numerator and the denominator, the COP coordinates are:

$$x = x_1 \{0.5 * \rho_0 + 0.33 * k_x * x_1 + 0.25 * k_y * y_1\} \{ \rho_0 + 0.5 * (k_x * x_1 + k_y * y_1) \}^{-1} \quad (vi)$$

$$y = y_1 \{0.5 * \rho_0 + 0.25 * k_x * x_1 + 0.33 * k_y * y_1\} \{ \rho_0 + 0.5 * (k_x * x_1 + k_y * y_1) \}^{-1} \quad (vii)$$

Note for a homogeneous slab with $k_x = k_y = 0$, the coordinate simplifies to $(0.5 * x_1, 0.5 * y_1)$, which is the centroid of the slab.

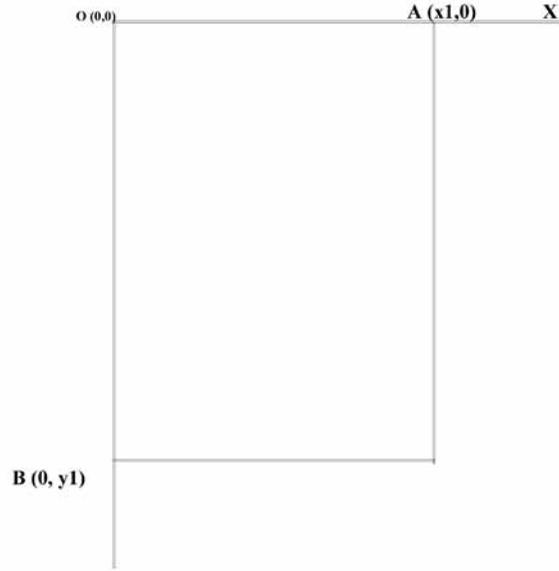


Figure 1. The rectangular thin slab of rock along with the chosen co-ordinate axes.

Case 2:

The vertical density variation can vary significantly from linearity, and can be represented as (Mukherjee, 2017):

$$\rho_{bwy} = \rho_m - (\rho_m - \rho_f)\phi_0 e^{-by} \tag{viii}$$

where

ρ_{bwy} : Bulk wet density of sediment at depth 'y'; ρ_m : density of matrix; ρ_f : density of pore fluid; ϕ_0 : surface porosity; b^{-1} : compaction constant.

Considering a linear variation of density along X-direction as in eqn (iii),

$$\rho(x,y) = k_x x + \rho_m - (\rho_m - \rho_f)\phi_0 e^{-by} \tag{ix}$$

For $x=y=0$, i.e. at origin, the density of $[\rho_m - (\rho_m - \rho_f)\phi_0]$ is considered.

Putting the expression of $\rho(x,y)$ into eqns (i) and (ii) and performing the definite integral in the numerator and the denominator, the COP coordinates are:

$$x = A B^{-1} \tag{x}$$

$$y = C B^{-1} \tag{xi}$$

Here

$$A = x_1^2 [0.5 * \rho_m y_1 - 0.5 * b^{-1} (\rho_m - \rho_f) \phi_0 (1 - e^{-by_1}) + 0.33 k_x x_1 y_1] \tag{xii}$$

$$B = x_1 [0.5 * \rho_m y_1^2 + (\rho_m - \rho_f) \phi_0 b^{-1} e^{-by_1} (y + b^{-1}) + 0.25 k_x x_1 y_1^2] \tag{xiii}$$

$$C = x_1 y_1 \{ \rho_m + 0.5 * k_x x_1 \} + (\rho_m - \rho_f) \phi_0 x_1 b^{-1} (e^{-by_1} - 1) \tag{xiv}$$

Note, here for $k_x=b=0$, i.e., for a homogeneous slab with density ρ_m , $x=0.5 * x_1$, and $y=0.5 * y_1$. This point is the center of gravity, which matches with our intuition.

DISCUSSIONS

Case 1 works for igneous intrusions, and case 2 especially for sedimentary slabs/dykes. In case density variation is present just along one direction, COP can still be deduced by taking one of the $k_i = 0$ (case 1), or by taking either $k_i = 0$ or $b = 0$ (case 2). The coordinate depends on the length and the width of the slice (x_1 and y_1), the density gradient (k_x, k_y), compaction constant (b^{-1}), surface porosity (ϕ_0), matrix density (ρ_m) and the fluid density (ρ_f). However no simple proportionality relation exists between the co-ordinate and any of these parameters. The present analysis (eqns i to xiv) holds true for very thin vertical igneous and sedimentary dykes, which do exist in nature (Alm and Sundbond 2002). The deductions will become approximate if the dip of the dyke is not perfectly vertical, or if the dyke is of several meters width (Fodor and Kazmer 1989). Thinner the dyke and steeper it dips, closer will be the match with the presented COP analysis. Vertical basalt dykes are common (Wellman and Wilson 1964) and mm-scale thin varieties are rare (Krumbholz et al., 2014). Sedimentary dykes passively fill up the space created by tectonic fractures (Roshoff and Cosgrove 2002). The present analysis works if the tectonic stresses cease and hydrostatic condition prevail in the sub-surface.

A three dimensional extension of the concept COP in terms of its coordinate (x,y,z) can be made by writing:

$$x = \frac{\int_0^{x_1} \int_0^{y_1} \int_0^{z_1} \rho(x,y,z) x \, dx \, dy \, dz}{\int_0^{x_1} \int_0^{y_1} \int_0^{z_1} \rho(x,y,z) \, dx \, dy \, dz} \tag{xv}$$

$$y = \frac{\int_0^{x_1} \int_0^{y_1} \int_0^{z_1} \rho(x,y,z) y \, dx \, dy \, dz}{\int_0^{x_1} \int_0^{y_1} \int_0^{z_1} \rho(x,y,z) \, dx \, dy \, dz} \quad (\text{xvi})$$

$$z = \frac{\int_0^{x_1} \int_0^{y_1} \int_0^{z_1} \rho(x,y,z) z \, dx \, dy \, dz}{\int_0^{x_1} \int_0^{y_1} \int_0^{z_1} \rho(x,y,z) \, dx \, dy \, dz} \quad (\text{xvii})$$

This is the same as the center of gravity of the rectangular parallelepiped with length z_1 along the Z-axis. Note that the densities of compacting sediments and cooling igneous materials will naturally be time dependent. Therefore, the presented model requires improvements to deal with such events over a relatively longer geological time period.

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Compliance with Ethical Standards

The author declares that he has no conflict of interest and adheres to copyright norms.

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